

Separable expansions for covariance estimation via the partial inner product

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Abstract

The non-parametric estimation of covariance lies at the heart of functional data analysis, whether for curve or surface-valued data. The case of a two-dimensional domain poses both statistical and computational challenges, which are typically alleviated by assuming separability. However, separability is often questionable, sometimes even demonstrably inadequate. We propose a framework for the analysis of covariance operators of random surfaces that generalises separability, while retaining its major advantages. Our approach is based on the expansion of the covariance into a series of separable terms. The expansion is valid for any covariance over a two-dimensional domain. Leveraging the key notion of the partial inner product, we generalise the power iteration method to general Hilbert spaces and show how the aforementioned expansion can be efficiently constructed in practice at the level of the surface observations. Truncation of the expansion and retention of the leading terms automatically induces a non-parametric estimator of the covariance, whose parsimony is dictated by the truncation level. The resulting estimator can be calculated, stored and manipulated with little computational overhead relative to separability. Consistency and rates of convergence are derived under mild regularity assumptions, illustrating the trade-off between bias and variance regulated by the truncation level. The merits and practical performance of the proposed methodology are demonstrated in a comprehensive simulation study. [Joint work with Tomas Masak and Victor M. Panaretos]